## Study of Hadronic Five-Body Decays of Charmed Mesons

## The FOCUS Collaboration

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## **Abstract**

We study the decay of  $D^+$  and  $D_s^+$  mesons into charged five-body final states, and report the discovery of the decay mode  $D^+ \to K^+ K^- \pi^+ \pi^+ \pi^-$ , as well as measurements of the branching ratios of the decay modes  $D^+ \to K^- \pi^+ \pi^+ \pi^+ \pi^-$ ,  $D_s^+ \to K^+ K^- \pi^+ \pi^+ \pi^-$ ,  $D_s^+ \to \phi \pi^+ \pi^+ \pi^-$  and  $D^+/D_s^+ \to \pi^+ \pi^+ \pi^+ \pi^-$ . An analysis of the resonant substructure for  $D^+ \to K^- \pi^+ \pi^+ \pi^+ \pi^-$  and  $D_s^+ \to K^+ K^- \pi^+ \pi^+ \pi^-$  is also included, with evidence suggesting that both decays proceed primarily through an  $a_1$  vector resonance.

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The hadronic five-body decays of charmed mesons have been studied in recent years [1–6], but limited statistics have prevented precise measurements of their resonant substructure. Theoretical predictions are limited mainly to two-body decay modes, and little is known about how five-body final states are produced. Theoretical discussion suggests a "vector-dominance model," in which heavy flavor mesons decay into a two-body intermediate state by emitting a W, which immediately hadronizes into a charged vector, axial vector, or pseudoscalar meson [7]. The charged meson then decays strongly to produce a many-body final state. Confirmation of this model could provide a mechanism for the production of five-body final states.

The FOCUS Collaboration [8–10] has studied two five-body decay modes,

 $D^+ \to K^- \pi^+ \pi^+ \pi^+ \pi^-$  and  $D_s^+ \to K^+ K^- \pi^+ \pi^+ \pi^-$ . We find evidence that in both modes the resonant substructure is dominated by a two-body vector resonance involving the  $a_1(1260)^+$ . We also present inclusive branching ratio measurements of four charged five-body hadronic decays, including the first evidence of the decay mode  $D^+ \to K^+ K^- \pi^+ \pi^+ \pi^-$ .

Five-body  $D^+$  and  $D_s^+$  decays are reconstructed using a candidate driven vertex algorithm [8]. A decay vertex is formed from the five reconstructed tracks. The momentum vector of the parent D meson is then used as a seed to intersect other tracks in order to find the production vertex. Events are selected based on a number of criteria. The confidence level of the decay vertex is required to be greater than 1%. The confidence level that a track from the decay vertex intersects the production vertex is required to be less than 1%. The likelihood for each particle to be a proton, kaon, pion, or electron based on Cerenkov particle identification is used to make additional requirements [9]. For each kaon candidate we require the negative log-likelihood kaon hypothesis,  $W_K = -2 \ln(\text{kaon likelihood})$ , to be favored over the corresponding pion hypothesis  $W_{\pi}$  by  $W_{\pi} - W_{K} > 3$ . In addition, for each pion candidate we require the pion hypothesis to be favored over any alternative hypothesis. We also require the significance of separation of the production and decay vertices to be at least 10. In order to reduce background due to secondary interactions of particles from the production vertex, we require the D reconstructed momentum to be greater than 25 GeV/c and the secondary vertex to be outside of the target. Finally, we remove events that are consistent with various  $D^*$ decays.

We turn now to additional analysis cuts made in individual modes, beginning with  $D^+ \to K^- \pi^+ \pi^+ \pi^+ \pi^-$ . Because this mode is the most abundant we apply only the standard cuts used in all modes. Figure 1a shows the  $K4\pi$  invariant mass plot. The distribution is fitted with a Gaussian for the  $D^+$  signal and a  $2^{\rm nd}$  degree polynomial for the background. A binned maximum likelihood fit gives  $2923 \pm 78$  events.

The  $D^+/D_s^+ \to \pi^+\pi^+\pi^+\pi^-\pi^-$  modes are more difficult to detect, due to the large combinatorial background. To reduce this background we increase the separation of the secondary vertex from the target to two standard deviations. We further impose a series of selection cuts to remove misidentified charm decays. We remove the decays  $D^+/D_s^+ \to \eta'\pi^+$ ,  $\eta' \to \pi^+\pi^+\pi^-\pi^-\pi^0$  by requiring the four pion reconstructed mass to be larger than the  $\eta' - \pi^0$  mass difference, that is,  $M_{4\pi} > 0.825 \,\text{GeV}/c^2$ . Figure 1b shows the five-pion invariant mass plot for events that satisfy these cuts. The distribution is fitted with a Gaussian for the  $D^+$  signal (835 ± 49 events), another Gaussian for the  $D_s^+$  signal (671 ± 47 events) and a 1<sup>st</sup> degree polynomial for the background.

For the  $D_s^+ \to K^+ K^- \pi^+ \pi^+ \pi^-$  mode the requirement of two kaons in the final

state greatly reduces background, allowing us to apply only the standard cuts used in all modes. Figure 1c shows the  $K^+K^-\pi^+\pi^+\pi^-$  invariant mass plot for events satisfying these cuts. We fit to a Gaussian (240  $\pm$  30 events) and 2<sup>nd</sup> degree polynomial.

For the  $K^+K^-\pi^+\pi^+\pi^-$  final state we have also studied the subresonant decay  $D_s^+ \to \phi \pi^+\pi^+\pi^-$ , by additionally requiring the  $K^+K^-$  invariant mass combination to be consistent with the  $\phi$  mass. The  $\phi \pi^+\pi^+\pi^-$  invariant mass plot is shown in Figure 1e. We fit to a Gaussian (136  $\pm$  14 events) and 2<sup>nd</sup> degree polynomial.

The decay  $D^+ \to K^+ K^- \pi^+ \pi^+ \pi^-$  is Cabibbo suppressed. We require a significance of vertex separation of 20, and tighten particle identification cuts on both kaons to  $W_\pi - W_K > 4$ , but remove all requirements on the pions. We also require the  $D^+$  reconstructed momentum to be greater than 50 GeV/c. Figure 1d shows the resulting  $K^+ K^- \pi^+ \pi^+ \pi^-$  invariant mass plot. This is the first observation of this mode. We fit with a Gaussian for the  $D^+$  signal (38±8 events), another Gaussian for the  $D_s$  events and a 2<sup>nd</sup> degree polynomial for the background.

We measure the branching fraction of the  $D^+ \to K^- \pi^+ \pi^+ \pi^- m$  mode relative to  $D^+ \to K^- \pi^+ \pi^+$ , then measure the branching fractions of the other  $D^+$  modes relative to the  $D^+ \to K^- \pi^+ \pi^+ \pi^+ \pi^-$  to reduce systematic effects due to differences in the number of decay products. All  $D_s^+$  decay modes are measured relative to  $D_s^+ \to K^+ K^- \pi^+$ . The normalizing decay modes are subjected to the same vertex cuts and analogous Čerenkov identification cuts as the mode in question to minimize systematic errors. The detector and analysis efficiency is calculated using a Monte Carlo simulation. For modes included in our resonant substructure analysis the Monte Carlo contains the incoherent mixture of subresonant decays determined by our analysis. For modes not included in our resonant substructure analysis, the Monte Carlo is composed of five-body phase space. We test for dependency on cut selection by individually varying each cut. The results, compared with existing measurements, are shown in Table 1.

We studied systematic effects due to uncertainties in Monte Carlo simulation, fitting procedure, resonant substructure, Monte Carlo statistics and absolute tracking efficiency. To determine the systematic error we follow a procedure based on the S-factor method used by the Particle Data Group [11]. For each mode we split the data sample into four independent subsamples based on D momentum and period of time in which the data was collected. We then define the split sample variance as the difference between the scaled variance and the statistical variance if the former exceeds the latter. We also evaluate systematic effects associated with Monte Carlo simulation of multi-body decays. The branching ratios are evaluated with multiple conditions on the isolation of the

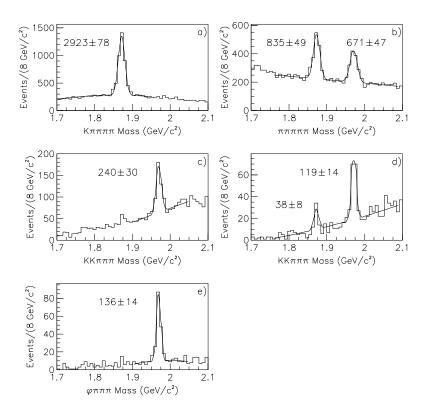


Fig. 1. (a)  $K4\pi$  invariant mass distribution. (b)  $5\pi$  invariant mass distribution. (c)  $KK3\pi$  invariant mass distribution for  $D_s^+$  optimized cuts. (d)  $KK3\pi$  invariant mass distribution for  $D^+$  optimized cuts. (e)  $\phi 3\pi$  invariant mass distribution. The fits are described in the text and the numbers quoted are the yields.

production vertex, with the variance used as the systematic error. In addition we evaluate the systematic effects based on different fitting procedures. The branching ratios are evaluated under various fit conditions, and the variance is used as the systematic error, as all fit variants are a priori equally likely. We also evaluate systematic effects due to uncertainty in resonant substructure by calculating the branching ratios using various mixtures of subresonant states in the Monte Carlo. The variance in the branching ratios for different subresonant mixtures is used as the systematic error, treating each subresonant mixture as a priori equally likely. We also evaluate the systematic effect from Monte Carlo statistics, adding in quadrature the uncertainty in the calculated efficiencies from both the signal and normalizing mode. Finally we evaluate systematic effects from uncertainty in absolute tracking efficiency of multi-body decays using studies of  $D^0 \to K^-\pi^+\pi^+\pi^-$  and  $D^0 \to K^-\pi^+$  decays. The systematic effects are then all added together in quadrature to obtain the final systematic error.

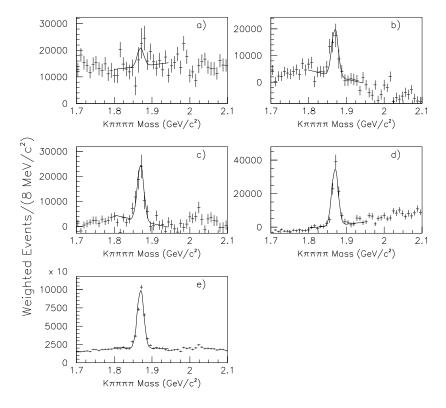
In addition to reporting inclusive branching ratio measurements, we have stud-

Table 1 Branching ratios for five-body modes and comparison to the previous measurements by E687. All branching ratios are inclusive of subresonant modes.

Decay Mode	FOCUS	E687[6]
$\frac{\Gamma(D^+ \to K^- \pi^+ \pi^+ \pi^+ \pi^-)}{\Gamma(D^+ \to K^- \pi^+ \pi^+)}$	$0.058 \pm 0.002 \pm 0.006$	$0.077 \pm 0.008 \pm 0.010$
$\frac{\Gamma(D^+ \to \pi^+ \pi^+ \pi^+ \pi^- \pi^-)}{\Gamma(D^+ \to K^- \pi^+ \pi^+ \pi^+ \pi^-)}$	$0.290 \pm 0.017 \pm 0.011$	$0.299 \pm 0.061 \pm 0.026$
$\frac{\Gamma(D_s^+ \to \pi^+ \pi^+ \pi^+ \pi^- \pi^-)}{\Gamma(D_s^+ \to K^- K + \pi^+)}$	$0.145 \pm 0.011 \pm 0.010$	$0.158 \pm 0.042 \pm 0.031$
$\frac{\Gamma(D_s^+ \to K^+ K^- \pi^+ \pi^+ \pi^-)}{\Gamma(D_s^+ \to K^- K^+ \pi^+)}$	$0.150 \pm 0.019 \pm 0.025$	$0.188 \pm 0.036 \pm 0.040$
$\frac{\Gamma(D_s^+ \to \phi \pi^+ \pi^+ \pi^-)}{\Gamma(D_s^+ \to \phi \pi^+)}$	$0.249 \pm 0.024 \pm 0.021$	$0.28 {\pm} 0.06 {\pm} 0.01$
$\frac{\Gamma(D^+ \to K^+ K^- \pi^+ \pi^+ \pi^-)}{\Gamma(D^+ \to K^- \pi^+ \pi^+ \pi^+ \pi^-)}$	$0.040\pm0.009\pm0.019$	

ied the resonance substructure in two decays:  $D^+ \to K^- \pi^+ \pi^+ \pi^-$  and  $D_s^+ \to K^+ K^- \pi^+ \pi^+ \pi^-$ . We use an incoherent binned fit method, also used in reference 6, a simplified approach which assumes the final state is an incoherent superposition of subresonant decay modes containing vector resonances. A coherent analysis for decays into five-body final states has not yet been attempted, and would be very difficult given the statistics of this experiment. For the  $D^+ \to K^- \pi^+ \pi^+ \pi^- \pi^-$  mode we consider the lowest mass  $(K^- \pi^+)$  and  $(\pi^+ \pi^-)$  resonances, as well as a nonresonant channel:  $\overline{K^{*0}}\pi^-\pi^+\pi^+$ ,  $K^- \rho^0\pi^+\pi^+$ ,  $\overline{K^{*0}}\rho^0\pi^+$ , and  $(K^-\pi^+\pi^+\pi^+\pi^-)_{\rm NR}$ . All states not explicitly considered are assumed to be included in the nonresonant channel.

We determine the acceptance corrected yield into each subresonant mode using a weighting technique whereby each event is weighted by its kinematic values in three submasses:  $(K^-\pi^+)$ ,  $(\pi^+\pi^-)$ , and  $(\pi^+\pi^+)$ . No resonance in the  $(\pi^+\pi^+)$  submass exists, but we include it in order to compute a meaningful  $\chi^2$ estimate of the fit. The weights are obtained using separate Monte Carlo simulations for the four decay modes, with the Particle Data Group values [11] for the mass and width of each resonance. Eight kinematic bins are constructed depending on whether each of the three submasses falls within the expected resonance (In the case of  $\pi^+\pi^+$ , the bin is split into high and low mass regions). For each Monte Carlo simulation the bin population in the eight bins is determined using a sideband subtracted cut on the  $D^+$  peak, allowing a linear transformation matrix to be calculated. The weights are then determined from the transformation matrix by a  $\chi^2$  minimization procedure. Each data event which satisfies our selection cuts is then weighted according to its kinematic values in the submass bins. Once the weighted distributions for each of the four modes are generated, we determine the acceptance corrected yield by fitting the distributions with a Gaussian signal and a linear background. Using incoherent Monte Carlo mixtures of the four subresonant modes we verified that our procedure was able to correctly recover the generated mixtures of the four modes.



<u>Fig.</u> 2.  $K^-\pi^+\pi^+\pi^+\pi^-$  weighted invariant mass for (a)  $(K^-\pi^+\pi^+\pi^+\pi^-)_{NR}$ , (b)  $\overline{K^{*0}}\pi^-\pi^+\pi^+$ , (c)  $K^-\rho^0\pi^+\pi^+$ , (d)  $\overline{K^{*0}}\rho^0\pi^+$ , (e) Inclusive sum of all four modes.

The results for  $K^-\pi^+\pi^+\pi^+\pi^-$  are summarized and compared to the E687 results in Table 2. Taking into account the correlation among the subresonant fractions, the calculated  $\chi^2$  for the hypothesis that the results are consistent with E687 is 6.5 (4 degrees of freedom). The four weighted histograms with fits are shown in Fig. 2. Figure 2e is the weighted distribution for the sum of all subresonant modes. The goodness of fit is evaluated by calculating a  $\chi^2$ for the hypothesis of consistency between the model predictions and observed data yields in each of the 8 submass bins. The calculated  $\chi^2$  is 7.4 (4 degrees of freedom), with most of the  $\chi^2$  contribution resulting from a poor Monte Carlo simulation of the  $\pi^+\pi^+$  spectrum for the  $\overline{K^{*0}}\rho^0\pi^+$  mode. We assessed systematic errors by individually varying the width of the submass bins corresponding to the  $\rho$  and  $\overline{K^{*0}}$  resonances by 20%. The systematic error is then estimated as the variance of the two measurements with varied widths, along with the original measurement. Since our methods of calculating subresonant fractions and inclusive branching ratios are distinct, statistical and systematic errors are added in quadrature when normalizing our subresonant fractions to other modes.

We follow a similar procedure for the  $D_s^+ \to K^+ K^- \pi^+ \pi^+ \pi^-$ , treating the

Table 2 Fractions relative to the inclusive mode and comparison to previous measurements for the resonance substructure of the  $D^+ \to K^- \pi^+ \pi^+ \pi^+ \pi^-$  decay mode. These values are not corrected for unseen decay modes.

Subresonant Mode	Fraction of $K4\pi$	E687 Fraction [6]
$(K^-\pi^+\pi^+\pi^+\pi^-)_{NR}$	$0.07 \pm 0.05 \pm 0.01$	< 0.26(90% C.L.)
$\overline{K^{*0}}\pi^-\pi^+\pi^+$	$0.21 \pm 0.04 \pm 0.06$	$0.42 {\pm} 0.14$
$K^-  ho^0 \pi^+ \pi^+$	$0.30 \pm 0.04 \pm 0.01$	$0.44 {\pm} 0.14$
$\overline{K^{*0}}\rho^0\pi^+$	$0.40 \pm 0.03 \pm 0.06$	$0.20 \pm 0.09$

final state as an incoherent superposition of the  $(K^+K^-)$  and  $(\pi^+\pi^-)$  resonances, as well as a nonresonant channel:  $\phi\pi^+\pi^+\pi^-$ ,  $K^+K^-\rho\pi^+$ ,  $\phi\rho\pi^+$  and  $(K^+K^-\pi^+\pi^+\pi^-)_{\rm NR}$ . Each event is weighted by its value in each of three submasses:  $(K^+K^-)$ ,  $(\pi^+\pi^-)$ , and  $(\pi^+\pi^+)$ , and the weighted distributions are again fitted with a Gaussian signal and a linear background. The results are summarized in Table 3 and are presented in Fig 3. The goodness of fit is evaluated by calculating a  $\chi^2$  for the hypothesis of consistency between the model predictions and observed data yields in each of the eight submass bins. The calculated  $\chi^2$  is 10.2 (4 degrees of freedom), with most of the  $\chi^2$  contribution resulting from a poor Monte Carlo simulation of the  $\pi^+\pi^+$  spectrum in the nonresonant channel. We assess systematic errors by calculating the variance of our results with 20% variations in the width of the submass bins corresponding to the  $\rho$  and  $\phi$  resonances.

In both resonant substructure analyses the dominant mode is of the form vector-vector-pseudoscalar:  $\overline{K^{*0}}\rho^0\pi^+$  and  $\phi\rho^0\pi^+$  in the case of  $K^-\pi^+\pi^+\pi^-\pi^-$  and  $K^+K^-\pi^+\pi^+\pi^-$ , respectively. Given the phase space constraints for both of these decays, such a result is unexpected. However, theoretical discussion of a vector-dominance model for heavy flavor decays [7] suggests that charm decays are dominated by quasi-two-body decays in which the  $W^{\pm}$  immediately hadronizes into a charged pseudoscalar, vector, or axial vector meson. Thus branching ratios of the form  $D \to a_1(1260)^+X$  are of comparable value to

Table 3 Fractions relative to the inclusive mode for the resonance substructure of the  $D_s^+ \rightarrow K^+K^-\pi^+\pi^+\pi^-$  decay mode. These values are not corrected for unseen decay modes.

Subresonant Mode	Fraction of $2K3\pi$
$(K^+K^-\pi^+\pi^+\pi^-)_{NR}$	$0.10 \pm 0.06 \pm 0.05$
$\phi\pi^-\pi^+\pi^+$	$0.21 \pm 0.05 \pm 0.06$
$K^+K^- ho^0\pi^+$	<0.03~(90%~C.L.)
$\phi  ho^0 \pi^+$	$0.75 \pm 0.06 \pm 0.04$

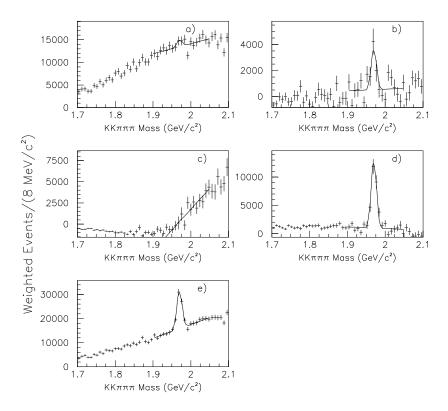


Fig. 3.  $K^+K^-\pi^+\pi^+\pi^-$  weighted invariant mass for (a)  $(K^+K^-\pi^+\pi^+\pi^-)_{NR}$ , (b)  $\phi\pi^-\pi^+\pi^+$ , (c)  $K^+K^-\rho^0\pi^+$ , (d)  $\phi\rho^0\pi^+$ , (e) Inclusive sum of all four modes.

those observed for  $D \to \pi^+ X$ , when adjusted for phase space. Such theoretical discussion raises the possibility that the resonant substructure for both modes is dominated by a quasi-two-body decay involving the  $a_1$ :  $\overline{K^{*0}}a_1^+$  and  $\phi a_1^+$  for  $K^-\pi^+\pi^+\pi^-$  and  $K^+K^-\pi^+\pi^+\pi^-$ , respectively, where  $a_1^+\to \rho^0\pi^+$ . Although the central value of the  $a_1$  mass lies outside of phase space for both decays, these decay modes are allowed due to the large width of the  $a_1$ . However, the large width of the  $a_1$  and its position in phase space make the resonance difficult to detect directly.

To verify the subresonant decays are proceeding through  $a_1$  we generate Monte Carlo simulations of  $D^+ \to \overline{K^{*0}} a_1^+$  and  $D_s^+ \to \phi a_1^+$ , assuming the  $a_1$  has a width of 400 MeV/ $c^2$  and decays entirely as an S-wave, and use our subresonant analysis procedure explained above. In both cases the yield fractions in each of the subresonant modes from the  $a_1$  Monte Carlo are similar to the reported fractions from the data, with particular agreement in the case of  $D_s^+ \to \phi a_1^+$ . Such agreement suggests both channels may be dominated by a two-body intermediate state involving the  $a_1$ .

Accepting the hypothesis that five-body modes are dominated by quasi-two-

body decays, we calculate branching ratios for the decays  $D^+ \to \overline{K^{*0}} a_1^+$  and  $D_s^+ \to \phi a_1^+$  using the ratios of the observed fractions of  $K^{*0} \rho \pi^+$  and  $\phi \rho \pi$  from data (40% and 75%) to the observed fractions from Monte Carlo simulations of  $D^+ \to \overline{K^{*0}} a_1^+$  and  $D_s^+ \to \phi a_1^+$  (70% and 78%). Assuming the  $a_1^+$  decays to  $\rho^0 \pi^+$  50% of the time and using the Particle Data Group  $\phi$  and  $K^{*0}$  branching fractions [11], the  $D^+ \to \overline{K^{*0}} a_1^+$  and  $D_s^+ \to \phi a_1^+$  branching fractions, including unseen decays, are shown in Table 4. We assess systematic errors by increasing the width of the  $a_1$  resonance in our generated Monte Carlo to 600 MeV/ $c^2$ , taking the systematic error as the variance of our measurements with the two widths.

Table 4 Inclusive branching ratios for  $a_1^+$  states. These values are corrected for unseen decay modes.

Decay Mode	Fraction
$\frac{\Gamma(D^+ \to \overline{K^{*0}} a_1^+)}{\Gamma(D^+ \to K^- \pi^+ \pi^+)}$	$0.099 \pm 0.008 \pm 0.018$
$\frac{\Gamma(D_s^+ \to \phi a_1^+)}{\Gamma(D_s^+ \to K^+ K - \pi^+)}$	$0.559 \pm 0.078 \pm 0.044$

In conclusion we have measured the relative branching ratios of five-body and three-body charged hadronic decays of  $D^+$  and  $D_s^+$  and have presented the first evidence of the decay mode  $D^+ \to K^+ K^- \pi^+ \pi^+ \pi^-$ . We have also performed an analysis of the resonant substructure in the decays  $D^+ \to K^- \pi^+ \pi^+ \pi^-$  and  $D_s^+ \to K^+ K^- \pi^+ \pi^+ \pi^-$ . Our analysis provides some evidence that both decays proceed through a quasi-two-body decay involving the  $a_1(1260)^+$  particle.

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